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INTERIOR PATH METHODS FOR HEURISTIC INTEGER PROGRAMMING PROCEDU--ETC(U)
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BY

BRUCE H. FAALAND and FREDERICK S. HILLIER

TECHNICAL REPORT NO. 73
FEBRUARY 1977

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DEPARTMENT OF OPERATIONS RESEARCH
STANFORD UNIVERSITY
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INTERIOR PATH METHODS FOR HEURISTIC INTEGER PROGRAMMING PROCEDURES

by

Bruce H. Faaland

and

Frederick S. Hillier

1. Introduction

Because of the significant computational limitations of integer programming algorithms for obtaining an optimal solution, there has been considerable research in recent years on heuristic algorithms for efficiently seeking very good solutions that are not guaranteed to be optimal. For example, such algorithms have been developed by Reiter and Rice [10], Echols and Cooper [1], Senju and Toyoda [12], Hillier [6,5], Roth [11], Kochenberger McCarl, and Wyman [8], Ibaraki, Ohashi, and Mine [7], and Toyoda [14]. Although their primary motivation was to provide an effective way of dealing with the frequently encountered integer programming problems that are beyond the computational capability of exact algorithms, heuristic algorithms also can be very useful on smaller problems by providing an advanced starting solution to accelerate an exact algorithm. For example, the availability of an advanced starting solution has been exploited very advantageously by the authors in Hillier's Bound-and-Scan algorithm [4] and Faaland and Hillier's Accelerated Bound-and-Scan algorithm [2].

Some of these heuristic algorithms are expressly designed for binary (0-1) integer programming while others deal with the general integer linear programming problem. The algorithms developed by Hillier [6,5] and extended by Ibaraki et al [7] fall into the latter category. One of their distinctive features is that the search for good integer solutions is focused in the neighborhood of the optimal solution for the corresponding linear programming problem (i.e., the original problem except for deleting integrality, restrictions) obtained by the simplex method. Specifically, the following three-phase approach is used. Phase 1 identifies a path leading from the optimal linear programming solution into the interior of the feasible region (when ignoring integrality restrictions). (Hillier originally proposed a linear path and Ibaraki et al extended this to a piecewise linear path.) In conceptual terms, Phase 2 then moves slowly along this path, using it as a "home base" from which to search for a nearby feasible (integer) solution. Phase 3 attempts to move from the feasible solution obtained to a succession of better ones. The final solution obtained is the desired approximate solution.

One device for attempting to improve further the quality of the approximate solution is to generate a number of distinct final solutions. The most effective means that Hillier [6,5] found to accomplish this is to repeat Phase 2 and 3 by moving down the path in Phase 2 beyond the point at which the previous Phase 2 solution had been found. He also found evidence to indicate that the location of the Phase 2 solution relative to the constraints may have a strong influence on the quality of the solution determined by Phase 3. That is, the degree of maneuverability allowed by the Phase 2 solution seems to be an important consideration. These two factors, substantial movement along the Phase 1 path and degree of maneuverability, indicate that the choice of a "home base" path to follow into the feasible region in Phase 2 deserves careful attention. The selection of this path is the subject of this paper.

2. Notation

The problem under consideration is to choose $x_1, x_2, \ldots, x_n$ so as to

$$(1) \qquad \text{maximize } Z = \sum_{j=1}^{n} c_j x_j,$$

subject to

$$(2) \qquad \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \qquad i = 1,2,\ldots,m$$

$$(3) \qquad x_j \geq 0, \qquad j = 1,2,\ldots,n$$

$$(4) \qquad x_j \text{ integer} \qquad j = 1,2,\ldots,n',$$

so $x_{n'+1}, \ldots, x_n$ need not be an integer. Let $\underline{x}$ be the n-vector $(x_1, x_2, \ldots, x_n)$, and let $\underline{x}^{(1)}$ be the optimal linear programming solution obtained by the simplex method by neglecting the integer constraint (4). Define

$$(5) \qquad B = \{j \mid j \leq n' \text{ and } x_j^{(1)} \text{ is a basic variable}\},$$

and let N be the number of elements of B. Given any $\underline{x}$, let $\underline{x}_B$ denote the corresponding N-vector whose components are x_j and that $j \in B$. Finally, let $d_i(\underline{x})$ be the Euclidean distance from $\underline{x}$ to the hyperplane $\sum_{j=1}^{n} a_{ij} x_j = b_i$, and when projecting from $\underline{x}$-space to $\underline{x}_B$-space, let $d_i(\underline{x}_B)$ be the Euclidean distance from $\underline{x}_B$ to the hyperplane $\sum_{j \in B} a_{ij} x_j = b_i - \sum_{j \notin B} a_{ij} x_j$. Thus (after introducing a sign convention),

$$(6) \qquad d_i(\underline{x}) = \left(b_i - \sum_{j=1}^{n} a_{ij} x_j\right) / \left(\sum_{j=1}^{n} a_{ij}^2\right)^{\frac{1}{2}}, \qquad i = 1,2,\ldots m$$

$$(7) \qquad d_i(\underline{x}_B) = \left(b_i - \sum_{j=1}^{n} a_{ij} x_j\right) / \left(\sum_{j \in B} a_{ij}^2\right)^{\frac{1}{2}}, \qquad i = 1,2,\ldots,m.$$

3. Review of Previous Methods

Hillier [6,5] originally proposed and tested two methods for choosing the interior path in Phase 1. Both methods are based on the following general approach. First, use the simplex method to obtain $\underline{x}^{(1)}$. Then, for each functional constraint i in (2) that is binding at $\underline{x}^{(1)}$ (i.e., whose slack variable is nonbasic for $\underline{x}^{(1)}$), replace b_i by

(8) $$b_i^{(2)} = b_i - \Delta_i$$

for some suitably chosen scalar $\Delta_i \geq 0$. For this modification of the linear programming problem (1-3), use standard post-optimality calculations to obtain the new solution $\underline{x}^{(2)}$ having the same basis as $\underline{x}^{(1)}$. Note that $\underline{x}^{(2)}$ also is feasible for the original linear programming problem (1-3) except perhaps for some of the constraints that are not binding at $\underline{x}^{(1)}$. Therefore, the objective of Phase 1 is fulfilled by letting the line segment between $\underline{x}^{(1)}$ and $\underline{x}^{(2)}$ be the desired interior path.

Method 1 sets

(9) $$\Delta_i = \frac{1}{2}\sum_{j \epsilon B} |a_{ij}|,$$

so that every basic variable that is restricted to be integer by (4) can be changed by as much as $\frac{1}{2}$ (as when rounding to the nearest integer value) without violating the original constraint i in (2).

Method 2 sets

(10) $$\Delta_i = \frac{1}{2}\left(\sum_{j=1}^{n} a_{ij}^2\right)^{\frac{1}{2}} N^{\frac{1}{2}},$$

so that

(11) $$d_i(\underline{x}^{(2)}) = \frac{1}{2}N^{\frac{1}{2}}.$$

These methods are illustrated in Figure 1 for a problem with functional constraints $4x_1 \leq 5$ and $2x_1 + 2x_2 \leq 5$, where the objective function is such that both of these constraints are binding at $\underline{x}^{(1)}$.

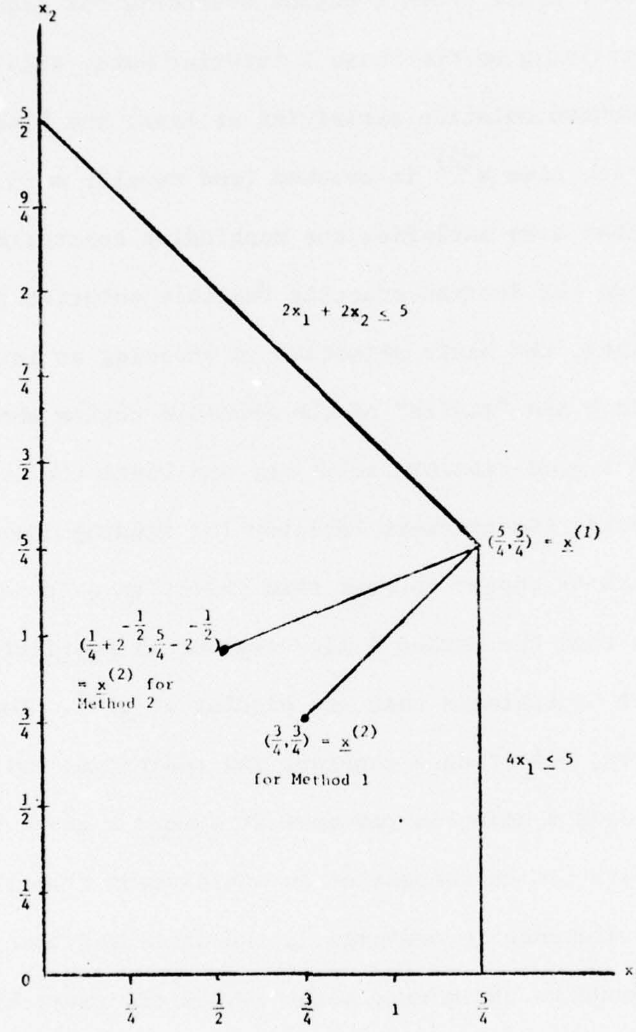

Figure 1. Interior paths for previous Phase 1 methods.

Both methods have the key property that every $\underline{x}_j^{(2)}$ such that $j \in B$ can be rounded to the nearest integer value without violating any of the original constraints (2-3) that are binding at $\underline{x}^{(1)}$. (This holds for Method 2 since the Euclidean distance between this rounded solution and $\underline{x}^{(2)}$ is at most $\frac{1}{2}N^{\frac{1}{2}}$.) Therefore, since Phase 2 begins searching for feasible solutions by rounding solutions lying on the Phase 1 interior path, this property guarantees that a rounded solution satisfying at least the binding constraints will be obtained by the time $\underline{x}^{(2)}$ is reached (and usually much sooner). If some such solution that also satisfies the nonbinding constraints can be found, it will provide the desired starting feasible solution for Phase 3.

Broadly speaking, the basic objective in choosing an interior path is [illegible] ave it lead into the "middle" of the feasible region where it should [illegible] st to locate a good feasible solution, and where the located feasible solutions should provide the greatest latitude for finding improved solutions in Phase 3. Both methods appear to meet this objective quite well. In fact, it follows from (11) that the Method 2 line segment is <u>equidistant</u> from the functional constraint hyperplanes that are binding at $\underline{x}^{(1)}$. Method 1 does not have this property, but it does consider the additional relevant question of just how far rounding a solution can move it <u>toward</u> a given hyperplane. Thus, the Method 1 path can be thought of as equidistant from the binding hyperplanes when the distance is measured in the direction that the maximum possible amount of rounding can move a point toward the given hyperplane.

The next section evaluates more critically just how well these methods actually meet this objective when it is defined more carefully.

4. Evaluation of Previous Methods

The two previous methods discussed in Section 3 both focus on extreme cases. What is the largest possible distance that rounding can move a solution? (Method 2) What is the largest possible increase in the left-hand side of a functional constraint that can result from rounding? (Method 1)

This raises the question of just how these extreme cases compare with the actual cases that normally would be occurring on a statistical basis. This is evaluated below, first in regard to the distances moved by rounding, and then the changes in left-hand sides. Implications are then explored.

To study this question statistically, assume that the parameters of the problem (1-4) are drawn from some underlying probability distributions. Assume, without loss of generality, that the ordering of the variables is such that $B = \{1,2,\ldots,N\}$. Then, for some given method of calculating the Δ_i needed to determine $\underline{x}^{(2)}$ (see Section 3), let the random variable X_j be the change in $x_j^{(2)}$ resulting from rounding it to the nearest integer, for $j = 1,2,\ldots,N$. Under most circumstances, one would expect the fractional parts of the $x_j^{(2)}$ to be dispersed rather uniformly over the unit interval. (For certain highly structured problems with integer-valued parameters, the X_j may have discrete distributions with considerable mass at zero. Such problems tend to be relatively easy to solve, however, and so are of less interest for heuristic procedures.) Therefore, it is assumed here that $X_1, X_2, \ldots, X_N$ are independent and identically distributed according to a uniform distribution ranging from $-\frac{1}{2}$ to $+\frac{1}{2}$, so that this common distribution has the density function

$$(12) \qquad f(x) = \begin{cases} 1, & \text{for } -\frac{1}{2} \le x \le \frac{1}{2} \\ 0, & \text{otherwise.} \end{cases}$$

Now let the random variable D be the Euclidean distance that $\underline{x}^{(2)}$ moves due to rounding $x_j^{(2)}$ to the nearest integer for $j = 1,2,\ldots,N$, so that

$$(13) \qquad D = \left(\sum_{j=1}^{N} X_j^2 \right)^{\frac{1}{2}}.$$

Theorem 1: As N grows large, the probability distribution of D tends to a normal distribution with mean and variance,

$$(14) \qquad E(D) = \left(\frac{N}{12}\right)^{\frac{1}{2}}, \quad \mathrm{Var}(D) = \frac{1}{60}\,.$$

Proof: Let the random variable Z have a normal distribution with mean zero and variance one, and let

$$(15) \qquad Z_N = \frac{\left(\sum_{j=1}^{N} X_j^2 \right)^{\frac{1}{2}} - \left(\frac{N}{12}\right)^{\frac{1}{2}}}{\left(\frac{1}{60}\right)^{\frac{1}{2}}}, \text{ for } N = 1,2,\ldots\,.$$

Thus, letting $\rightarrow$ denote convergence of a sequence of random variables in the sense of pointwise convergence of their cumulative distribution functions, it is sufficient to show that

$$Z_N \rightarrow Z.$$

It is easily calculated that

$$(16) \qquad E(X_j^2) = \frac{1}{12}\,, \ \mathrm{Var}\,(X_j^2) = \frac{1}{180}\,.$$

Therefore, by the Central Limit Theorem,

$$\frac{D^2 - \frac{N}{12}}{\left(\frac{N}{180}\right)^{\frac{1}{2}}} \to Z.$$

Now construct a Taylor series expansion of the square root of D^2 about its mean, $E(D^2) = \frac{N}{12}$,

$$(17) \qquad D = \left(\frac{N}{12}\right)^{\frac{1}{2}} + \frac{1}{2}\left(\frac{N}{12}\right)^{-\frac{1}{2}}\left(D^2 - \frac{N}{12}\right) - \frac{1}{8}\xi^{-\frac{3}{2}}\left(D^2 - \frac{N}{12}\right)^2$$

for some ξ interior to the interval joining D^2 and $\frac{N}{12}$ (so ξ is a function of the value taken on by D^2). Consequently,

$$(18) \qquad Z_N = \frac{D - \left(\frac{N}{12}\right)^{\frac{1}{2}}}{\left(\frac{1}{60}\right)^{\frac{1}{2}}} = \frac{D^2 - \frac{N}{12}}{\left(\frac{N}{180}\right)^{\frac{1}{2}}} - \varepsilon_N,$$

where

$$\varepsilon_N = \frac{1}{8}(60)^{\frac{1}{2}}\left(\frac{N}{180}\right)\xi^{-\frac{3}{2}}\left[\frac{D^2 - \frac{N}{12}}{\left(\frac{N}{180}\right)^{\frac{1}{2}}}\right]^2,$$

so

$$1440(60)^{-\frac{1}{2}}N^{-1}\xi^{\frac{3}{2}}\varepsilon_N \to Z^2.$$

Now note that

$$N^{-1}\xi^{\frac{3}{2}} = \frac{1}{12}\left(\frac{N}{12}\right)^{\frac{1}{2}}\left[1 + \frac{\xi - \frac{N}{12}}{\frac{N}{12}}\right]^{\frac{3}{2}} \to \infty \qquad \text{with probability one,}$$

since

$$\left|\frac{\xi - \frac{N}{12}}{\left(\frac{N}{180}\right)^{\frac{1}{2}}}\right| < \left|\frac{D^2 - \frac{N}{12}}{\left(\frac{N}{180}\right)^{\frac{1}{2}}}\right| \to |Z|$$

implies

$$\frac{\xi - \frac{N}{12}}{\frac{N}{12}} \to 0 \qquad \text{with probability one.}$$

Therefore,

$$\varepsilon_N \to 0 \qquad \text{with probability one,}$$

so

$$Z_N \to Z. \qquad \text{Q.E.D.}$$

Although Theorem 1 gives asymptotic results, these also can be used as approximations for even small values of N.
The Central Limit Theorem usually will give a reasonably good approximation to the normal distribution for values of N as small as four or five. Furthermore, an analysis of additional terms in the above Taylor series expansion (17) indicates that the expresssions for E(D) and Var(D) given by Theorem 1 should be within a few percentage points of the true values for

values of N this small. This implies that Var(D) is essentially a constant independent of N, even though E(D) essentially is growing proportionally to $N^{\frac{1}{2}}$.

It is interesting to compare the range of <u>possible</u> values of D,

$$(19) \qquad 0 \leq D \leq \frac{1}{2}N^{\frac{1}{2}}$$

with the range of <u>likely</u> values suggested by Theorem 1. For example, using the mean $\pm$ nearly four standard deviations gives

$$(20) \qquad \text{Prob } \{(\frac{N}{12})^{\frac{1}{2}} - \frac{1}{2} \leq D \leq (\frac{N}{12})^{\frac{1}{2}} + \frac{1}{2}\} > 0.9998.$$

Thus, even though Method 2 focuses on the upper bound, $D \leq \frac{1}{2}N^{\frac{1}{2}}$, for all practical purposes one can instead assume that $D \leq (N/12)^{\frac{1}{2}} + \frac{1}{2}$. Furthermore, for fairly large values of N, the actual values taken on by D will tend to be quite close to $3^{-\frac{1}{2}} \cong 58\%$ times the upper bound used by Method 2.

Even more pertinently, Method 2 uses $\frac{1}{2}N^{\frac{1}{2}}$ as an upper bound on the distance that rounding a solution can move it <u>toward</u> a given functional constraint hyperplane, and Method 1 uses a similar quantity. By contrast, D represents the distance moved in <u>whichever</u> direction results from the rounding, so the resulting decrease in the distance to the hyperplane is likely to be much smaller than D. Therefore, a realistic statistical upper bound on this decrease may be far smaller than the upper bounds used by these methods.

To study this question, let the random variable D_i and D_i' ($i = 1,2,\ldots,m$) be the decrease in $d_i(\underline{x})$ and $d_i(\underline{x}_B)$, respectively, due to first setting $\underline{x} = \underline{x}^{(2)}$ and then rounding x_j to the nearest integer for $j = 1,2,\ldots,N$. Thus

$$(21) \qquad D_i = \frac{\sum_{j=1}^{N} a_{ij} X_j}{\left(\sum_{j=1}^{n} a_{ij}^2\right)^{\frac{1}{2}}}, \qquad D_i' = \frac{\sum_{j=1}^{N} a_{ij} X_j}{\left(\sum_{j=1}^{N} a_{ij}^2\right)^{\frac{1}{2}}},$$

so

$$D_i = C D_i', \quad \text{where } C = \left[\frac{\sum_{j=1}^{N} a_{ij}^2}{\sum_{j=1}^{n} a_{ij}^2}\right]^{\frac{1}{2}}, \ 0 \le C \le 1.$$

Theorem 2: (a) $E(D_i) = 0$, $\qquad E(D_i') = 0$

$$(22) \qquad \text{Var}(D_i) = \frac{C^2}{12}, \qquad \text{Var}(D_i') = \frac{1}{12}$$

$$\text{Prob}\,\{D_i \le k\frac{C}{\sqrt{12}}\} \ge 1 - \frac{1}{k^2}, \qquad \text{Prob}\,\{D_i' \le k\frac{1}{\sqrt{12}}\} \ge 1 - \frac{1}{k^2}$$

for any $k > 1$.

(b) As N grows large, if $\{a_{i1}^2, a_{i2}^2 \ldots\}$ necessarily is a uniformly bounded sequence such that its sum diverges, then the probability distribution of D_i and D_i' tends to a normal distribution with the respective means and variances given above.

<u>Proof:</u> Part (a) follows immediately by direct calculation of the moments and then Chebychev's inequality. Part (b) is a direct application of the Lindeberg condition[1] for the Central Limit Theorem.

The condition on the a_{ij} parameters in Theorem 2b is not a particularly restrictive one. For example, it holds if these coefficients for a given constraint can be thought of as being independently drawn from a single underlying probability distribution that is bounded but not degenerate at zero. However, the rate at which the distribution of D_i and D_i' approaches a normal distribution depends considerably on the dispersion of the a_{ij}^2. If a very few coefficients dominate the others in magnitude, then the normal approximation will not yet be a good one. But if there are many relatively large coefficients of the same order of magnitude, it will be an excellent approximation.

If N is large and the normal approximation is reasonable, then the statistical upper bounds become very small indeed compared to the bounds used by Methods 1 and 2. For example, using the mean plus $\sqrt{12}$ standard deviations,

$$\text{(23)} \qquad \text{Prob}\{D_i \leq C\} > 0.9997, \qquad \text{Prob}\{D_i' \leq 1\} > 0.9997.$$

---

1 See Feller [3, pp. 256-258].

Note that neither bound ever exceeds one regardless of how large N is. By contrast, Method 2 uses the upper bound,

$$(24) \qquad D_i \le \frac{1}{2} N^{\frac{1}{2}},$$

and Method 1 uses

$$(25) \qquad D_i \le \frac{\frac{1}{2} \sum_{j=1}^{N} |a_{ij}|}{\left(\sum_{j=1}^{n} a_{ij}^2\right)^{\frac{1}{2}}} .$$

(Observe that (24) and (25) give the same upper bound if N = n and the a_{ij} are equal, but that (24) is larger otherwise.) Thus, these latter bounds can be many times too large to reflect realistic possibility, and so lose their relevance for their intended purpose.

The implication is that each Δ_i in (8) can sometimes be made much smaller in order to obtain an appropriate $\underline{x}^{(2)}$, e.g., by using (23) to set

$$(26) \qquad \Delta_i = C\left(\sum_{j=1}^{n} a_{ij}^2\right)^{\frac{1}{2}} = \left(\sum_{j=1}^{N} a_{ij}^2\right)^{\frac{1}{2}} .$$

Then, if one wants to use $\underline{x}^{(2)}$ directly to obtain a feasible solution by rounding (bypassing Phase 2), a far superior solution may result. Furthermore, this solution is far less likely to violate any of the constraints (2,3) that are not binding at $\underline{x}^{(1)}$ than are the rounded solutions resulting from the larger Δ_i in (9) and (10).

However, if the full three-phase procedure is to be used, then the real significance of Δ_i is not the resulting $\underline{x}^{(2)}$, but rather the direction of the line segment leading from $\underline{x}^{(1)}$ toward $\underline{x}^{(2)}$. That is, two methods for setting the Δ_i that differ by only a fixed multiplicative constant (for a given problem) actually would be equivalent in this respect since the direction would be the same. This is the case, for example, for the Δ_i in (10) and the Δ_i suggested by the statistical upper bound in (20). However, note that the Δ_i in (26) are not equivalent to the Δ_i in either (9) or (10), so (26) does give a new direction for the line segment.

It is enlightening to reflect on the differences between (26) and (10). At first glance, they appear to be quite similar since, except for (10) having the multiplicative constant given in (11), they differ only in the limit of summation being n for (10) and N for (26). However, this minor symbolic difference masks two major methodological differences. First, (10) gives as much influence to the integer-restricted variables (4) that are nonbasic at $\underline{x}^{(1)}$ $(x_{N+1}, x_{N+2}, \ldots, x_{n'})$ as to those that are basic $(x_1, x_2, \ldots, x_N)$, even though these nonbasic variables are irrelevant for rounding solutions between $\underline{x}^{(1)}$ and $\underline{x}^{(2)}$ (and play little role thereafter in Phase 2). Second, (10) also gives as much influence to the variables that are not integer-restricted $(x_{n'+1}, x_{n'+2}, \ldots, x_n)$, even though these variables play only a secondary role at best in any versions of Phase 2 developed to date. The search for a good feasible solution in Phase 2 takes place primarily in $\underline{x}_B$-space (using the orthogonal projection onto this coordinate hyperplane from the parallel hyperplane passing through the current point of interest on the line segment between $\underline{x}^{(1)}$ and $\underline{x}^{(2)}$). Therefore, these differences suggest some major drawbacks in Method 2, which is based on (10).

To illustrate these drawbacks, suppose that $n = 3$ for the problem with $N = 2$ shown in Figure 1, where the additional variable x_3 enters only into the $4x_1 \leq 5$ constraint as $4x_1 + 40x_3 \leq 5$. This changes $\underline{x}_B^{(2)}$ for Method 2 to $\underline{x}_B^{(2)} = (\frac{5}{4} - \frac{1}{2}\sqrt{202}, \frac{1}{4} + \frac{1}{2}\sqrt{202}) \simeq (-5.86, 7.36)$, so that the line segment from $\underline{x}_B^{(1)}$ now intersects the x_2-axis slightly above $x_2 = \frac{9}{4}$. The result is that the essentially irrelevant variable x_3 has greatly distorted the direction of the Phase 2 search for a good feasible solution by moving it far away from the center of the feasible region in $\underline{x}_B$-space.

Therefore, it appears to be more appropriate to focus on distances in $\underline{x}_B$-space, such as the D_i' variables considered above, rather than distances in $\underline{x}$-space. This conclusion leads to the new method presented in the next section.

5. A New Method

Based on the above analysis, it is proposed that $\underline{x}^{(2)}$ be obtained by making it equidistant in $\underline{x}_B$-space from the functional constraint hyperplanes that are binding at $\underline{x}^{(1)}$. Thus, the rationale is the same as for Method 2, except that the $d_i(\underline{x}_B)$ quantities in (7) now are equated rather than the $d_i(\underline{x})$ in (6). To be completely analogous to (10) and (11) for Method 2, one would set

$$\Delta_i = \frac{1}{2}\left(\sum_{j \in B} a_{ij}^2\right)^{\frac{1}{2}} N^{\frac{1}{2}}, \tag{27}$$

so that

$$\text{(28)} \qquad d_i(\underline{x}_B^{(2)}) = \frac{1}{2} N^{\frac{1}{2}} .$$

This would guarantee that $x_j^{(2)}$ can be rounded to the nearest integer for every $j \in B$ and the resulting solution would still satisfy all of the constraints that are binding at $\underline{x}^{(1)}$. However, (23) indicates that (28) often is overly conservative, since

$$\text{(29)} \qquad d_i(\underline{x}_B^{(2)}) = 1$$

can virtually provide the same guarantee (assuming the conditions of Theorem 2b hold). As already observed in Section 4, this statistical upper bound on the D_i leads to

$$\text{(26)} \qquad \Delta_i = (\sum_{j \in B} a_{ij}^2)^{\frac{1}{2}}.$$

With either (27) or (26), $\underline{x}^{(2)}$ still would be calculated by the procedure described at the beginning of Section 3.

Note that (26) and (27) actually provide equivalent methods (hereafter labeled Method 3) since, for any given problem, they differ only by the fixed multiplicative constant $\frac{1}{2} N^{\frac{1}{2}}$ and so give the same direction for the line segment from $\underline{x}^{(1)}$ to $\underline{x}^{(2)}$. Nevertheless, when Theorem 2b is applicable, it is suggested that (26) be used, since its $\underline{x}^{(2)}$ provides a more realistic estimate of the maximum amount by which the line segment should need to be extended from $\underline{x}^{(1)}$. In the rare instances where rounding this $\underline{x}^{(2)}$ (for $j \in B$) violates a binding constraint, the line segment always can be extended a little further. However, when this rounded solution is not feasible, the usual explanation would be that it violates one or more of the constraints that are not binding at $\underline{x}^{(1)}$, so it would do no good to extend the search further in the direction of the line segment. Therefore, failure to find a feasible solution by this time would be a good signpost that the search needs to be moved in another direction to take into account these relevant nonbinding constraints. A procedure for doing this is described in the next section.

Another reason for preferring (26) over (27) is its strong superiority for the following new option for streamlining Phase 2. Begin (as usual) by checking whether rounding $\underline{x}^{(1)}$ (for $j \in B$) provides a feasible solution. If not, then immediately check whether rounding $\underline{x}^{(2)}$ (for $j \in B$) provides a feasible solution. Only if this fails also would Phase 2 be resumed in the usual way. (Another possibility would be to conduct the usual Phase 2 search from both of these rounded solutions before considering any other points on the line segment between $\underline{x}^{(1)}$ and $\underline{x}^{(2)}$.) The rationale is that if rounding $\underline{x}^{(1)}$ (and perhaps searching from there) fails, then identifying a nearby feasible solution may be difficult, leading to a long Phase 2 process. By using the "tight" $\underline{x}^{(2)}$ provided by (26), much time might be saved by skipping down to where a feasible solution may be readily identified, while perhaps sacrificing little in the quality of this solution (or of the resulting final solution yielded by Phase 3).

6. Extension of Methods to Piecewise Linear Interior Paths

All of the methods discussed above focus on the functional constraints (2) and the nonnegativity constraints (3) that are binding at $\underline{x}^{(1)}$ (i.e., such that the slack variable in (2i) or the variable in (3j) is nonbasic for $\underline{x}^{(1)}$) by requiring that rounding $\underline{x}^{(2)}$ (for $j \in B$) should satisfy these constraints. There is no requirement, however, that either $\underline{x}^{(2)}$ or the corresponding rounded solution must (with even high probability) satisfy all of the constraints (2,3) that are not binding at $\underline{x}^{(1)}$. If the rounded solution does violate any of these constraints, then it becomes possible (but by no means certain) that Phase 2 will fail to find a feasible solution in the general region between $\underline{x}^{(1)}$ and $\underline{x}^{(2)}$. Furthermore, if any of the variables among $\underline{x}$ or the slack variables added to (2) are degenerate basic variables at $\underline{x}^{(1)}$, the line segment from $\underline{x}^{(1)}$ to $\underline{x}^{(2)}$ may lead immediately out of the feasible region. Although even this may not prevent finding a feasible solution, either complication would likely blunt the effectiveness of the otherwise highly successful(optional) technique proposed by Hillier [6,5] of generating multiple final solutions in Phase 3 by generating a series of feasible solutions in which each succeeding feasible solution corresponds to a new point further along the line segment from $\underline{x}^{(1)}$ to $\underline{x}^{(2)}$. These considerations suggest that the choice of an interior path to follow during Phase 2 preferably should be governed by the location of the path relative to all constraints of the problem, not only those which are binding at $\underline{x}^{(1)}$.

An attractive alternative choice of this kind for the interior path is the parametric solution to the linear program,

(30) maximize r,

subject to

(31) $$\sum_{j=1}^{n} a_{ij}x_j + \Delta_i r \leq b_i, \qquad i = 1,\ldots,m$$

(32) $$\sum_{j=1}^{n} c_j x_j = Z$$

(33) $$x_j \geq 0, \qquad j = 1,\ldots,n$$

(34) $$r \geq 0,$$

as Z is decreased from its value at $\underline{x}^{(1)}$. Thus this extension of Methods 1, 2, and 3 (labeled 1P, 2P, and 3P, where P stands for "Parametric") would use their respective definitions of Δ_i, namely, (9), (10), and (26)(or (27)). Because of the properties of parametric linear programming, the resulting path for each method is continuous and piecewise linear, leading from $\underline{x}^{(1)}$ into the interior of the feasible region for the continuous version of the original problem (1-3).

Since there no longer is a fixed set of variables (corresponding to B) that need rounding along such a path, one could adjust the Δ_i each time this set changes by redefining B and N in terms of the current basis instead of $\underline{x}^{(1)}$. This is not recommended, since it would considerably complicate the computations, introduce discontinuities into the path, and perhaps create chain reactions of changes. However, the price being paid for fixing the Δ_i should be recognized. Since the current values of such redefined B and N may indeed change with each new basis in the parametric solution, the original rationale for choosing the fixed Δ_i may no longer be completely applicable. Fortunately, this doesn't really affect Method 2P, since its Δ_i (10) is uninfluenced by B and N except for the same multiplicative factor $N^{\frac{1}{2}}$ for all i, which only changes the scale of r. However, B does play significant role in (9) (Method 1P) and in (26) (Method 3P). The relative amount by which changes in B would change these Δ_i depends greatly on the size of N and the dispersion of the a_{ij}. With N large, and with a_{ij} of comparable magnitude for different $j \in B$, any deviations between the Δ_i for the "current" B and the fixed Δ_i at $\underline{x}^{(1)}$ should be of little consequence.

If N is not large, or if a few a_{ij} dominate the others in magnitude, then it is recommended that the integer-restricted variables (4) that are nonbasic at $\underline{x}^{(1)}$ be fixed at zero in (30-34) for Methods 1P and 3P. This would ensure that the original rationale for (9) and (26) would continue to hold (conservatively) along the entire path, since it would restrict the rounding of $\underline{x}$ to just $\underline{x}_B$. The drawback is that it decreases the flexibility in choosing an appropriate interior path in $\underline{x}$-space.

It should be noted that Ibaraki et al [7] first proposed this approach for generating a piecewise linear interior path in the context of Method 2. (However, they also acknowledge [7,p.134] the independent development of this approach by the present authors.)

The piecewise linear interior path generated by each of these three methods follows a "most interior" route into the feasible region for (1-3) in the sense that, for every value of Z, the corresponding point along the route maximizes the minimum Euclidean distance to the boundary hyperplanes for the functional constraints (2), when this distance is measured in the appropriate way. For Method 1P, the distance to the hyperplane for constraint i should be measured from the point in the direction of the n-vector $\underline{d}$ such that $d_j = +1$ if $j \in B$ and $a_{ij} > 0$, $d_j = -1$ if $j \in B$ and $a_{ij} < 0$, and $d_j = 0$ otherwise. For Method 2P, the distance is measured in the usual way perpendicularly to the hyperplane, as in (6). For Method 3P, it is done in the same way as for 2P except in $\underline{x}_B$-space rather than $\underline{x}$-space, as given by (7). Thus, for Methods 2P and 3P, the point along the route for a given value of Z is the center of the largest sphere which may be inscribed within the functional constraints in the appropriate space, subject to the side conditions that the center must be nonnegative and lie on the hyperplane (32). When Method 3P uses (26), the radius of the sphere in $\underline{x}_B$-space is the optimal value of r defined by Z. This also would be the radius in $\underline{x}$-space for Method 2P if the unnecessary multiplicative factor, $\frac{1}{2}N^{\frac{1}{2}}$, were eliminated from (10).

The paths for Methods 2P and 3P are also motivated by the observation that every sphere in Euclidean N-space of radius $r \geq \frac{1}{2}N^{\frac{1}{2}}$ contains at least one lattice point, namely, the point which results from rounding the center of the sphere. (N here should be interpreted as the <u>current</u> number of variables that need rounding.)

Suppose r* is the radius of the largest sphere with nonnegative center which may be inscribed within the functional constraints in $\underline{x}$-space. The center of the sphere $\underline{x}^*$ lies along the Method 2P interior path. Furthermore, if $r^* \geq \frac{1}{2}(n')^{\frac{1}{2}}$, the heuristic procedure is guaranteed to find a feasible solution by proceeding along this interior path, since at worst, it will round $\underline{x}^*$ to find a feasible integer solution. These same remarks also apply to Method 3P in $\underline{x}_B$-space, with n' replaced by N (as defined in Section 2), if the integer-restriced variables (4) that are nonbasic at $\underline{x}^{(1)}$ have been fixed at zero in (30-34). Furthermore, when the conditions of Theorem 2b hold, (23) indicates that $r^* \geq C$ (for Method 2P) or $r^* \geq 1$ (for Method 3P) usually would be sufficient to find a feasible solution.

Another interesting property of the path generated by Method iP(i = 1,2,3) is that, barring degeneracy, its first segment leading away from $\underline{x}^{(1)}$ lies along the Method i line segment. The straightforward proof of this statement follows from the correspondence between the optimal basis to problem (1-3) and the initial basis in the parametric problem (30-34). This initial basis to (30-34) will have as basic variables r and all variables basic at $\underline{x}^{(1)}$. As Z is decreased, this basis remains optimal until one of the basic variables is driven to zero at some critical value of Z. At this point the basis changes and the interior path diverges from the corresponding Method i line segment.

7. Another Extension to Piecewise Linear Interior Paths

The above approach to generating piecewise linear interior paths does require a software package that includes parametric programming, as well as the considerable execution time involved. If this is not convenient, then the following is a simpler and quicker approach that only requires the basic simplex method needed to obtain $\underline{x}^{(1)}$ anyway.

First, apply the simplex method to solve the following variation of (1-3),

(35) $$\text{maximize } r,$$

subject to

(36) $$\sum_{j=1}^{n} a_{ij}x_j + \Delta_i r \le b_i, \qquad i = 1,2,\ldots,m$$

(37) $$x_j \ge 0, \qquad j = 1,2,\ldots,n,$$

starting with the initial solution $\underline{x}^{(1)}$. Record the sequence of basic feasible solutions generated in the process of doing this, and then connect each successive pair of these solutions (in $\underline{x}$-space) by a line segment. This is the desired piecewise linear path.

As before, this extension of Methods 1, 2, and 3 (labeled 1S, 2S, and 3S, where S stands for "Simplex") would use their respective definitions of Δ_i, namely, (9), (10), and (26) (or (27)).

Although the path generated by Method iS (i = 1,2,3) leads to the same "most interior" point (according to the respective distance norms described in Section 6) as for Method 1P, it does not follow a "most interior" route in the same sense. However, it does provide an alternate route to this "most interior" point which never leaves the feasible region for (1-3).

8. Computational Experience

In order to evaluate and compare these techniques, several FORTRAN codes were developed for a CDC-6400 system. Since Phase 1 of the heuristic algorithm developed by Ibaraki et al [7] is based on Method 2 (and since programming services were no longer available when Method 3 was developed), the focus was on testing variations of Method 2 (2, 2P, and 2S), although Method 1 (without extension) also was run on the problems.

A total of 64 test problems were used. 22 of these are standard test problems in the literature - Haldi's IBM problems (#1-5,9) and Fixed Charge problems (all 10), and Woolsey's 4-point and 5-point combinatorial problems (all 18 are reproduced by Trauth and Woolsey [15]), plus two problems (#4,5) given by Petersen [9] and two (#8,9) given by Thompson [13]. The other 42 are randomly generated problems that were previously used by the authors. These consist of the Type I problems (#1-8, and "Large" 30 x 60, 60 x 30, 60 x 60), Type II problems (#1-16), and Type III problems (#2-5,8) discussed in [6], as well as the Type V problems (#1-10) discussed in [2]. Of particular interest are the type I and II problems, whose a_{ij} parameters are randomly generated integers from the intervals, [-40,59]] and [0,99], respectively, and whose variables are general integer variables (as opposed to binary variables). All 64 problems were treated as pure integer programs, so n' = n.

Since the relevant consideration in evaluating Phase 1 methods is the resulting effectiveness of the overall heuristic procedure, these methods were applied to the test problems in conjunction with fixed Phases 2 and 3. These latter phases used Methods 3A and 1 respectively, according to the labels described in [6]. (The one minor exception is that Method 2A for Phase 2 was used in conjunction with Method 1 of Phase 1.) Phase 2 also used the device discussed in both [6] and [5] (and labeled R3A in [5]) of generating multiple feasible solutions for Phase 3. (This involves repeating the Phase 2 search all along the interior path generated by Phase 1, and so provides a better test of this path.) A maximum of 20 feasible solutions (excluding any obtained directly from $\underline{x}^{(1)}$) were allowed from Phase 2, where these solutions need not be distinct. The algorithm proceeded to Phase 3 with each new Phase 2 solution only if that solution differs from the immediately preceding ones and this was allowed at most 10 times.

Procedures 2-R3A-1, 2P-R3A-1, and 1-R2A-1 were applied to all 64 test problems. For 47 of these problems, they all found their best solution from the Phase 2 feasible solution generated directly from $\underline{x}^{(1)}$ so that the interior path generated by Phase 1 was irrelevant in these cases. For the other 17 test problems, the results regarding the best final solution (including the point in the algorithm at which this solution was found) are shown in Table 1 for Procedures 2-R3A-1 and 2P-R3A-1. (Procedure 1-R2A-1 is excluded from the table since a detailed comparison with Procedure 2-R2A-1 already is

Table 1

Performance of Methods 2, 2P, and 2S on Test Problems Where

Interior Path Needed to Obtain Best Solution

				Procedures 2-R3A-1				Procedure 2P-R3A-1				Procedure 2S-R3A-1			
PROBLEM	m	n	Optimal Z	Best Z	PH1	PH2	PH3	Best Z	PH1	PH2	PH3	Best Z	PH1	PH2	PH3
Petersen #5	10	28	1260	1170	1	1	1	1260	2	7	5	1220	1	2	2
Fixed Charge #9	6	6	9	9	1	2	2	9	1	2	2				
Thompson #9	10	7	-464	-468	1	1	1	-468	1	1	1				
Type I #2	15	15	2570	2565	1	3	2	2565	1	3	2	2570	1	3	3
Type I #4	15	15	2527	2482	1	6	3	2482	1	6	3				
Type I #5	15	15	6171	6133	1	1	1	6133	1	1	1	6133	1	1	1
Type I #6	15	15	2234	2207	1	4	4	2207	1	4	4	2188	1	1	1
Type I #8	15	15	2543	2516	1	1	1	2516	1	1	1				
Type II #1	15	15	$\geq$1875	1835	1	1	1	1875	2	13	7				
Type II #3	15	15	1983	1983	1	3	2	1983	1	3	2				
Type II #4	15	15	2429	2429	1	4	3	2429	2	6	3				
Type II #9	15	15	1743	1743	1	2	2	1743	1	2	2				
Type II #13	15	15	1785	1747	1	1	1	1747	1	1	1	1785	1	2	2
Type II #14	15	15	2309	2309	1	3	2	2285	2	3	2				
Type I Large	30	60	?	4486	1	4	2	4486	1	4	2	4419	1	8	5
Type I Large	60	30	?	2591	1	2	2	2591	1	2	2	2557	1	6	3
Type I Large	60	60	?	2457	1	3	3	2456	2	7	4	2446	2	12	7

Best Z = value of objective function for best final solution found by corresponding procedure.
Ph 1 = number of line segments already generated in Phase 1 when best final solution was found.
Ph 2 = number of feasible solutions (not necessarily distinct) already generated in Phase 2 when best final solution was found.
Ph 3 = number of times Phase 3 already entered when best final solution was found.

given in Table III of [6] for most of these same problems.) Procedure 2S-R3A-1 was applied to 7 of these 17 problems, as also shown in Table I.

Ibaraki et al [7] also applied their heuristic algorithm to 9 of these 64 test problems, with the comparative results shown in Table II (Thompson = T, Fixed Charge = FC). Times are given for applying the simplex method to find $\underline{x}^{(1)}$ (labeled LP), and then for the total of this and all three phases of the heuristic procedure per se. However, it should be noted that they used a different computer, a FACOM 230/60, which they indicate corresponds very roughly to the IBM 360/65 and the UNIVAC 1108. Furthermore, although they also generated multiple solutions on these problems (except for FC-7 and FC-10), they only allowed four distinct feasible solutions from Phase 2 and only used the two best of these for Phase 3.

Table II

Comparison with Ibaraki-Ohashi-Mine Algorithm

Problem	m	n	Optimal Z	Procedure 2P-R3A-1			Ibaraki-Ohashi-Mine Algorithm			Procedure 1-R2A-1			Procedure 2-R3A-1		
					Time (in secs)			Time (in secs)			Time (in secs)			Time (in secs)	
				Best Z	LP	Total	Best Z	LP	Total	Best Z	LP	Total	Best Z	LP	Total
T-9	10	7	-464	-468	.21	0.94	-468	.1	2.6	-468	.21	0.34	-468	.21	0.90
FC-7	4	5	76	76	.14	0.53	76	.01	0.3	76	.14	0.30	76	.14	0.46
FC-10	10	12	17	17	.14	0.68	17	.1	0.5	17	.14	0.40	17	.14	0.69
I-5	15	15	6171	6133	.34	2.12	6133	.2	4.0	6133	.34	1.54	6133	.34	1.88
I-6	15	15	2234	2207	.34	2.07	2231	.2	3.7	2234	.34	1.05	2207	.34	1.72
II-1	15	15	1875	1875	.24	4.29	1864	.1	4.3	1835	.24	0.60	1835	.24	1.03
II-3	15	15	1983	1983	.37	3.38	1983	.2	2.5	1983	.37	0.82	1983	.37	1.03
II-11	30	15	1491	1488	.51	4.29	1488	.1	4.0	1488	.51	1.05	1488	.51	1.05
II-12	30	15	1424	1424	.69	4.46	1424	.2	4.1	1424	.69	2.36	1424	.69	2.12

9. Conclusions

Under some circumstances, Method 1 and (particularly) Method 2 for Phase 1 can yield an exceedingly conservative $\underline{x}^{(2)}$. More seriously, Method 2 gives undue weight to largely irrelevant variables, and so can distort the resulting direction of the line segment from $\underline{x}^{(1)}$. Statistical analysis suggests a promising new Method 3 which avoids these shortcomings.

All of these methods focus only on the functional constraints that are binding at $\underline{x}^{(1)}$. Although this usually should be adequate, there are situations where other constraints should influence the interior path. The P (Section 6) and S (Section 7) extensions to piecewise linear interior paths provide this capability. Furthermore, the P extension starts off on the line segment for the original method (barring degeneracy), and so should provide a comparable performance on ordinary problems as well. The S extension may be more convenient, but its other properties are less certain.

On the 64 test problems, the P extension led to a different best solution from the original method on only four of them (better on two, slightly worse on two), so it certainly doesn't provide a dramatic improvement. However, it is only designed to help on those occasional problems where additional constraints should be considered, without hurting on others, and it does seem to meet this objective. Furthermore, it should increase the reliability of heuristic procedures in finding any feasible solution on problems where this is very difficult. Since the price for adding this extension is a modest one, it should prove to be a worthwhile addition for at least full-fledged production codes.

On the other hand, the S extension often gives a different best solution (frequently worse but sometimes better) from either the P extension or the original method. It apparently tends to start off the interior path in a much different direction. Therefore, its main usefulness may be as a supplementary method to obtain additional final solutions to try to improve upon the original best solution from another method.

Computational comparisons of the heuristic procedures tested here with the one proposed by Ibaraki et al [7] suggest that they have similar performance capabilities.

The reader is referred to [5] for an extensive comparison of Methods 1 and 2 (without extension), as well as some new options for all three phases and comprehensive experimental results on various versions of these heuristic procedures.

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